Toward a Foundation of Manufacturing Science:
Algorithms & Implementation for a Computer-Aided Setup and Inspection System

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Outline

1. Motivation
2. Geometric algorithms for workpiece localization
3. Tolerance formulation and verification
4. Performance & reliability analysis
5. Sampling and probe radius compensation
6. Implementation and applications
7. Conclusion
1. Motivation

In 1952, MIT Servo Lab (G.S. Brown) developed, in collaboration with Parsons, the first CNC milling machine.

- Manual machine with an operator
- Giddings & Lewis 5-axis Skin Miller (1957)
- Kearney & Trecker NC Turning
- Fujitsu & Fraice NC Mill (1958)
• In 1959, APT was developed, followed by extensive activities in CAD

Setup & Fixturing

CAD → CAM → CNC

(AutoCAD, ...) (UGII, MasterCam, ...) (Fanuc, Siemens, ...)

Pictorial representations of the various stages of the MIT numerical control project.
• Conventional Approaches

  - Jigs, fixtures & hard gauges:
    → Expensive!

  - Manual Setup:
    → Time consuming & expensive
1.1 A Computer-aided Setup System

CAD/CAM Data

Arbitrarily place & fixture workpiece with general purpose fixtures (Robots and/or programmable fixtures)

Probe and measure point data from the workpiece surfaces

Compute the location & orientation of the workpiece

Modify & optimize tool path with computed transformation

CNC machining

Parts
Outline

1. Motivation
2. Geometric algorithms for workpiece localization
   2.1 The problem
   2.2 The configuration spaces – a geometric view
   2.3 Problem formulation
   2.4 Analytic results
   2.5 Performance evaluation
   2.6 Symmetric localization
   2.7 Discrete symmetry
   2.8 The hybrid algorithm
2. Geometric algorithms for workpiece localization.

2.1 The Problem

Possible Geometries:
(a) Regular Workpiece  Regular localization
(b) Symmetry        Symmetric localization
(c) Partially machined  Hybrid localization/envelopment
(d) Raw stock        Envelopment
2.2 Configuration Spaces: A Geometric View

(a) Regular workpiece and the Euclidean group SE(3):

• Rotational Motion

\[ SO(3) = \{ R \in \mathbb{R}^{3 \times 3} \mid R^T R = I, \det R = 1 \} \]

\[ so(3) = \{ \hat{\omega} \in \mathbb{R}^{3 \times 3} \mid \hat{\omega}^T = -\hat{\omega} \} \cong \mathbb{R}^3 \]

\[ \hat{\omega} = \begin{bmatrix}
0 & -w_3 & w_2 \\
w_3 & 0 & -w_1 \\
w_3 & w_1 & 0
\end{bmatrix} \]

\[ \exp : \quad so(3) \rightarrow SO(3) : \quad \hat{\omega} \rightarrow e^{\hat{\omega}} = R \]

\[ \omega \in \mathbb{R}^3 : \quad \text{Exponential coordinates of } R \]
• General rigid motion

\[ SE(3) = \{(p, R) | p \in \mathbb{R}^3, R \in SO(3)\} \]

\[ g = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \in SE(3) \quad : \text{Euclidean group of } \mathbb{R}^3 \]

\[ se(3) = \{ \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{4 \times 4} | \omega, v \in \mathbb{R}^3 \} \quad : \text{Lie Algebra of } SE(3) \]

\[ \hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix}, \quad \xi = \begin{bmatrix} v \\ \omega \end{bmatrix} \]

\text{Exp : } \quad se(3) \rightarrow SE(3) \quad : \quad \hat{\xi} \rightarrow e^{\hat{\xi}} \quad : \text{Screw motion}
(b) Symmetric workpiece and homogeneous space

\[ \theta : \quad SE(3) \times \mathcal{L} \mapsto \mathcal{L} \]

\[ ((R, p), l_{(x_0, v)}) \mapsto l_{(Rx_0 + p, R v)} \]

- Transitive action

- Isotropy subgroup:

\[ G_{l(0, e_3)} = \left\{ \begin{bmatrix} e^{\lambda_1 e_3} & \lambda_2 e_3 \\ 0 & 1 \end{bmatrix} \middle| \lambda_1, \lambda_2 \in \mathbb{R} \right\} \]

- If \( l' = g_0 l \Rightarrow G_{l'} = g_0 G_{l} g_0^{-1} \)

- \( F : SE(3) / G_0 \mapsto \mathcal{L} : gG_0 \mapsto gl_0 \)

- Configuration Space:

\[ SE(3) / G_0 = \{ gG_0 | g \in SE(3) \} \]

\[ g_1 \sim g_2 \text{ If } g_1 \cdot g_2^{-1} \in G_0 \]

Elements of \( SE(3) / G_0 \) : [g], \( gG_0 \) or g.
A plane:

\[ G_0 = \left\{ \begin{bmatrix} \lambda \hat{e}_3 \\ \lambda_2 e_1 + \lambda_3 e_2 \\ 0 \end{bmatrix} | \lambda_1, \lambda_2 \in \mathbb{R} \right\} \]
<table>
<thead>
<tr>
<th>Features</th>
<th>Symbols</th>
<th>Isotropy Subgroup $G_0$</th>
<th>Configuration Space $Q$</th>
<th>Description of Isotropy Subgroup</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle in $\mathbb{E}^2$</td>
<td>![Circle]</td>
<td>$\text{SO}(2)$</td>
<td>$SE(2)/G_0 = T(2)$</td>
<td>Rotation around the center of circle</td>
<td>2</td>
</tr>
<tr>
<td>Straight line in $\mathbb{E}^2$</td>
<td>![Line]</td>
<td>$T(1)$</td>
<td>$SE(2)/G_0$</td>
<td>Translation along the line</td>
<td>2</td>
</tr>
<tr>
<td>Sphere</td>
<td>![Sphere]</td>
<td>$\text{SO}(3)$</td>
<td>$SE(3)/G_0 = T(3)$</td>
<td>Rotation about the center of sphere</td>
<td>3</td>
</tr>
<tr>
<td>Plane</td>
<td>![Plane]</td>
<td>$SE(2)$</td>
<td>$SE(3)/G_0$</td>
<td>Rotation about the normal of the plane &amp; translation in the plane</td>
<td>3</td>
</tr>
<tr>
<td>Straight line in $\mathbb{E}^3$</td>
<td>![Line]</td>
<td>$\text{SO}(2) \times T(1)$</td>
<td>$SE(3)/G_0$</td>
<td>Rotation about and translation along the line</td>
<td>4</td>
</tr>
<tr>
<td>Cylinder</td>
<td>![Cylinder]</td>
<td>$\text{SO}(2) \times T(1)$</td>
<td>$SE(3)/G_0$</td>
<td>Rotation about and translation along the axis</td>
<td>4</td>
</tr>
<tr>
<td>Cone (resolved surface)</td>
<td>![Cone]</td>
<td>$\text{SO}(2)$</td>
<td>$SE(3)/G_0$</td>
<td>Rotation about the axis of cone</td>
<td>5</td>
</tr>
<tr>
<td>Tabular Surface</td>
<td>![Tabular Surface]</td>
<td>$T(1)$</td>
<td>$SE(3)/G_0$</td>
<td>Translation along the sweeping direction</td>
<td>5</td>
</tr>
<tr>
<td>Sculptured Surface without symmetry</td>
<td>![Sculptured Surface]</td>
<td>$I$</td>
<td>$SE(3)$</td>
<td>Identity element</td>
<td>6</td>
</tr>
</tbody>
</table>
Configuration Space:

\[ SE(3)/G_0 = \{ gG_0 \mid g \in SE(3) \} \]

If \( g_1 \sim g_2 \) if \( g_1 \cdot g_2^{-1} \in G_0 \).

Elements of \( SE(3)/G_0 \) : \([g] \), \( gG_0 \) or \( g \).

Prop.: \( SE(3)/G_0 \) is a differentiable manifold of dimension \( \dim(SE(3)) - \dim(G_0) \), with a transitive action.

\[ \mu : SE(3) \times SE(3)/G_0 \rightarrow SE(3)/G_0 \]

\( (h, gG_0) \rightarrow hgG_0 \)
- Canonical Coordinates:

\[ g_0 : \text{Lie algebra of } G_0 \]
\[ M_0 \oplus g_0 = se(3) \]

**Define:**

**Exp:** 
\[ M_0 \oplus g_0 \to SE(3) \]
\[ (\hat{m}, \hat{h}) \to e^{\hat{m}} \cdot e^{\hat{h}} \]

Let \((\hat{\xi}_1, \ldots, \hat{\xi}_r)\) be a basis of \(M_0\), and

\[ m = y_1 \hat{\xi}_1 + \ldots + y_r \hat{\xi}_r \]

\[ \tilde{\Psi} : SE(3)/G_0 \to \mathbb{R}^r \]
\[ g \mapsto (y_1, \ldots, y_r) \]

is well defined, and provide a canonical coordinate system for \(SE(3)/G_0\)
• If \( m_l \oplus g_l \) is a decomposition for \( G/G_l \), and \( l' = g_0 l_0 \), \( \Rightarrow G_{l'} = g_0 G_l g_0^{-1} \)

\[ \Rightarrow g_{l'} = \text{Ad}_{g_0}(g_{l_0}) \]

\[ m_{l'} = \text{Ad}_{g_0}(m_l) \]
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   2.7 Discrete symmetry
   2.8 The hybrid algorithm
2. 3 Problem Formulation

(a) Regular Localization

Data : \( \{y_i\}_{i=1..n} \)
Find \( g \in \text{SE}(3), x_i \in S_i \)

\[
\min \varepsilon(g, x_1, ..., x_n) = \sum_{i=1}^{n} \|g^{-1}y_i - x_i\|^2
\]

(b) Symmetric Localization :

Find \( g \in \text{SE}(3)/G_0 \) s.t.

\[
\min \varepsilon(g, x_1, ..., x_n) = \sum_{i=1}^{n} \|g^{-1}y_i - x_i\|^2
\]

(c) Hybrid Localization/Envelopment Problem :

\( \{y_i\}_{i=1..n} \) : Finished surface with \( G_0 \)
\( \{z_i\}_{i=1..m} \) : Unmachined surface
Find \( g_0 \in SE(3)/G_0, \ x_i \in S_i \) s.t.
\[
\min \varepsilon (g) = \sum_{i=1}^{n} <g^{-1}y_i - x_i, n_i>^2
\]

Let
\[
g(\lambda) = g_0 G_0(\lambda)
\]

Find \( g(\lambda) \in SE(3), \ \omega_j \in S_j \) s.t.
\[
\min \varepsilon (\omega) = \sum_{j=1}^{m} <g^{-1}(\lambda)z_j - \omega_j, n_j>^2
\]
and
\[
<g^{-1}(\lambda)z_j - \omega_j, n_j> \geq \delta_j, \quad j = 1, \ldots, m
\]
Outline

1. Motivation
2. Geometric algorithms for workpiece localization
   2.1 The problem
   2.2 The configuration spaces – a geometric view
   2.3 Problem formulation
   2.4 Analytic results
   2.5 Performance evaluation
   2.6 Symmetric localization
   2.7 Discrete symmetry
   2.8 The hybrid algorithm
2. 4 Analytic Results

Define:

\[ \overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \quad \overline{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \]

\[ x'_i = x_i - \overline{x} \quad y'_i = y_i - \overline{y} \]

\[ W = \sum_{i} y'_i x'_i^T = U \Sigma V^T \quad (SVD) \]

\[ \Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & \sigma_3 \end{bmatrix} \]

**Thm:** If \( \text{Rank}(W) = 3 \) (i.e. \( n \geq 4 \)), \( \exists \! (R^*, p^*) \) minimize \( \varepsilon(g, x_1, \ldots, x_n) \) and

\[ \begin{cases} R^* = VU^T \\ p^* = \overline{x} - R^* \overline{y} \end{cases} \]

\[ \varepsilon^* = \sum_{i} \| x'_i \|^2 + \sum_{i} \| y'_i \|^2 - 2 \sum_{i} \sigma_i \]

\[ \varepsilon(g, x_1, \ldots, x_n) = \sum_{i=1}^{n} \| y_i - gx_i \|^2 \]
Proof: $\epsilon(R, p, \cdot) = \sum_{i} \|g y_i - x_i\|^2$

$$= n \|R y + p - \bar{x}\|^2 + \sum_{i} \|R y_i' - x_i'\|^2$$

$\Rightarrow p^* = \bar{x} - R^* \bar{y}$

$\epsilon(R) = \sum_{i} \|R y_i' - x_i'\|^2$

$$= \sum_{i} (\|y_i'\|^2 + \|x_i'\|^2) - 2 \sum_{i} <R y_i', x_i'>$$

$$= a - 2 \text{tr}(R W)$$

$$\hat{\omega} = \begin{bmatrix} 0 & -w_3 & w_2 \\
 w_3 & 0 & -w_1 \\
 w_3 & w_1 & 0 \end{bmatrix}$$

$\hat{\omega} R \in T_R SO(3)$
<d\varepsilon_R, \hat{\omega}R> = \frac{d}{dt}|_{t=0} \varepsilon(e^{t\hat{\omega}R})
= -2tr(\hat{\omega}RW) = 0, \forall \omega
\Rightarrow RW \text{ Symmetric}

Let

\[ RW = S \]
\[ \Rightarrow S^2 = W^T \cdot W \]

\[ S = (W^T \cdot W)^{1/2} = V \begin{bmatrix} \pm \sigma_1 & & \\ & \pm \sigma_2 & \\ & & \pm \sigma_3 \end{bmatrix} V^T \]

\[ \varepsilon^*(\cdot) = \sum (||y_i'||^2 + ||x_i'||^2) - 2tr(W^TW)^{1/2} \]

\[ i \]
\[ \Rightarrow \begin{cases} R^* = VU^T \\ p^* = \bar{x} - R^* \bar{y} \end{cases} \]
**Prop:** A necessary condition for $x_i^*, i = 1, \ldots, n$, to minimize $\varepsilon(R, p, \cdot)$ is that

\[
\begin{align*}
(x^i &= \Psi^i(u_i, v_i)) \\
\begin{cases}
<x'_i - g^{-1}y_i, \Psi_{u_i}^i> &= 0 \\
<x'_i - g^{-1}y_i, \Psi_{v_i}^i> &= 0
\end{cases} & \text{ for } i = 1, \ldots, n
\end{align*}
\]

where $\Psi^i : \mathbb{R}^2 \rightarrow \mathbb{R}^3$, parametric equation of $S_i$, and

\[
\bar{\varepsilon}(R, p) = \sum_{i} ||g^{-1}y_i - x_i^*||^2 = \sum_{i} <g^{-1}y_i - x_i^*, n>^2
\]
Localization Algorithms:

\[(g^0, x_i^0) \rightarrow (g', x_i') \rightarrow \cdots (g^*, x_i^*)\]

1) Variational Algorithm:

\[
\begin{align*}
R &= VU^T \\
\mathbf{p} &= \mathbf{x} - R\mathbf{y}
\end{align*}
\]

2) Tangent Algorithm:

\[
g^k = g^{k-1}e^\xi \approx g^{k-1}(I + \hat{\xi}), \quad \hat{\xi} = \begin{bmatrix} \hat{\omega} \\ v \\ 0 \end{bmatrix}, \quad \omega, v \in \mathbb{R}^3
\]

Find \( \xi = \begin{bmatrix} v \\ \omega \end{bmatrix} \) by

\[
\text{min } \varepsilon(\xi) = \sum_{i} \left\| (g^k)^{-1} y_i - x_i \right\|^2
\]

\[
A \cdot \xi = b
\]

\[
A = \begin{bmatrix}
\sum_{i=1}^{n} \hat{y}_{ik}^{-1} & -\sum_{i=1}^{n} \hat{y}_{ik}^{-1} \cdot \hat{y}_{ik}^{-1} \\
-\sum_{i=1}^{n} \hat{y}_{ik}^{-1} \cdot \hat{y}_{ik}^{-1} & -\sum_{i=1}^{n} \hat{y}_{ik}^{-1} \cdot \hat{y}_{ik}^{-1}
\end{bmatrix} \in \mathbb{R}^{6 \times 6},
\]

\[
b = \begin{bmatrix}
-\sum_{i=1}^{n} \hat{y}_{ik}^{-1} \times x_i^k \\
-\sum_{i=1}^{n} (x_i^k - y_{ik}^{k-1})
\end{bmatrix} \in \mathbb{R}^{6}
\]
3) Hong-Tan Algorithm:

\[ g^k = g^{k-1}e^{\hat{x}} \approx g^{k-1}(I+\hat{x}) \]

Find \( \hat{\xi} = \begin{bmatrix} \nu \\ \omega \end{bmatrix} \) by

\[
\min \varepsilon(\hat{\xi}) = \sum_{i} <(g^k)^{-1}y_i-x_i, n_i>^2,
\]

\[
\bar{A}\cdot\hat{\xi} = \bar{b}
\]

\[
\bar{A} = \begin{bmatrix}
\sum_{i=1}^{n}(\hat{y}_i^{k-1}\times n_i^k)(n_i^k)^T & -\sum_{i=1}^{n}(\hat{y}_i^{k-1}\times n_i^k)(\hat{y}_i^{k-1}\times n_i^k)^T \\
\sum_{i=1}^{n}n_i^k(n_i^k)^T & \sum_{i=1}^{n}n_i^k(\hat{y}_i^{k-1}\times n_i^k)^T
\end{bmatrix} \in \mathbb{R}^{6 \times 6},
\]

\[
\bar{b} = \begin{bmatrix}
-\sum_{i=1}^{n}(\hat{y}_i^{k-1}\times n_i^k)(\hat{y}_i^{k-1}\times x_i^k, n_i^k) \\
-\sum_{i=1}^{n}(\hat{y}_i^{k-1}\times x_i^k, n_i^k)n_i^k
\end{bmatrix} \in \mathbb{R}^{6}
\]
Algorithm (Alternating Variable Method)

Input: \( Y = \{ y_i \}_{i=1}^n, \ y_i \in S_i \)

Step 0:
1. Set \( k = 0 \);
2. Initialize \( g^0 \);
3. Compute \( y_i^0 = (g^0)^{-1} y_i \);
4. Compute \( x_i^0 \);
5. Compute \( \varepsilon^0 = \varepsilon(g^0, x^0) \);
6. \( k = k + 1 \).

Step 1:
1. Newton's algorithm for \( x_i^k \);
2. Compute \( g^k \) using \( (x_i^k, g^{k-1}) \);
3. Compute \( y_i^k = (g^k)^{-1} y_i \);
4. Compute \( \varepsilon^k = \varepsilon(g^k, x^k) \);
5. If \( (1 - \varepsilon^k / \varepsilon^{k-1}) < \delta_1 \), exit
   else
6. Set \( k = k + 1 \), return to Step 1(a)
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1. Motivation
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   2.1 The problem
   2.2 The configuration spaces – a geometric view
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   2.7 Discrete symmetry
   2.8 The hybrid algorithm
2.5 Performance Evaluation

**Algorithms:**
- (a) Variational Algorithm
- (b) ICP Algorithm
- (c) Tangent Algorithm
- (d) Meng’s Algorithm
- (e) Hong-Tan Algorithm

**Performance Criteria:**
- (a) Robustness
- (b) Accuracy
- (c) Efficiency
Regions of convergence in terms of the maximal orientation errors for each of the algorithms

Accuracy of estimation achieved by each of the algorithms as a function of the number of measurement points
Computational efficiency by each of the algorithms as a function of the number of measurement points

**Summary:**

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Robustness</th>
<th>Accuracy</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hong-Tan</td>
<td>Good(-20 ^\circ \sim 20 ^\circ)</td>
<td>highest</td>
<td>highest</td>
</tr>
<tr>
<td>Variational</td>
<td>better (-30 ^\circ \sim 30 ^\circ)</td>
<td>high</td>
<td>high</td>
</tr>
<tr>
<td>Tangent</td>
<td>Best (-60 ^\circ \sim 60 ^\circ)</td>
<td>high</td>
<td>high</td>
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1. Motivation
2. Geometric algorithms for workpiece localization
   2.1 The problem
   2.2 The configuration spaces – a geometric view
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   2.4 Analytic results
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   2.7 Discrete symmetry
   2.8 The hybrid algorithm
2.6 Symmetric Localization

Find \( g \in SE(3)/G_0, \ x_i \in F \) to

\[
\text{minimize} \quad \varepsilon(g, x_1, ..., x_n) = \sum_{i=1}^{n} ||y_i - gx_i||^2
\]

Choose

\[
M_0 \oplus g_0 = se(3) \quad \text{and} \quad M_0 = \text{span}\{\xi_1, ..., \xi_k\}
\]

Algorithm: (Symmetric Localization)

Input:
- a) Measurement data \( \{y_i\}_{i=1..n} \)
- b) CAD description of \( F \)

Output:
- Optimal solution \( g^* \in SE(3)/G_0, x_i \in F \)
Step 0:

a) Set $k=0$;
b) Initialize $g_0$;
c) Solve for $x_i^0$, $i=1, \ldots, n$;
d) Calculate $\varepsilon_0 = \sum_i \|y_i - g_i x_i^0\|^2$

Step 1:

a) Let $g_{k+1} = e^{\hat{m}} g_k$, $\hat{m} \in \text{Ad}_{g_k} (M_0)$
   Solve for $\hat{m}$ by minimizing
   
   $\varepsilon(\hat{m}) = \sum_i \|y_i - g_{k+1} x_i^k\|^2$
   
   or
   
   $\varepsilon(\hat{m}) = \sum_i <g_{k+1}^{-1} y_i - x_i^k, n_i^k>^2$

b) Solve for $x_i^{k+1}$
c) Calculate $\varepsilon_{k+1}$
d) If $(1 - \varepsilon_{k+1}/\varepsilon_k) > \varepsilon$, set $k = k+1$;
   go to step 1(a). Else exit.
Example: A plane in $\mathbb{R}^3$

$$x(u,v) = u e^1 + v e^2$$

$$G_0 = \begin{bmatrix} \lambda_1 \hat{e}_3 & \lambda_2 e_1 + \lambda_3 e_2 \mid \lambda_i \in \mathbb{R} \end{bmatrix}$$

$$g_0 = \text{span}\{\hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_6\}$$

$$M_0 = \text{span}\{\hat{\xi}_3, \hat{\xi}_4, \hat{\xi}_5\}$$

Simulation results for a plane in $\mathbb{R}^3$
Performance Evaluation:

• Robustness with respect to initial conditions

Forbes
TSL
FSL

Efficiency comparison

Forbes
TSL
FSL
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   2.2 The configuration spaces – a geometric view
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   2.8 The hybrid algorithm
2. 7 Discrete Symmetry

- Composite Feature:

\[
G_A = \{e^{m_1\xi_1 + m_2\xi_3 + m_3\xi_5} \mid m_1, m_2, m_3 \in R\}
\]
\[
G_B = \{e^{m_1\xi_2 + m_2\xi_3 + m_3\xi_4} \mid m_1, m_2, m_3 \in R\}
\]
\[
G_C = \{e^{m_1\xi_1 + m_2\xi_2 + m_3\xi_6} \mid m_1, m_2, m_3 \in R\}
\]

\[\Rightarrow G_{ABC} = G_A \cap G_B \cap G_C = I\]

Q: a unique solution?
**Plane:**

\[ G_0 = SE(2) \times D, \quad D = \{1, -1\} \]

Identify configurations differing by \( G_0 \)

**Cube:**

\[ G_{ABC} = G_A \cap G_B \cap G_C = \{I, e^{\pi^\xi_4}, e^{\pi^\xi_5}, e^{\pi^\xi_6}\} \]

• **Solution:**

Filter out solutions with deviating home point

• **Remark:**

\[
A\xi = b, \quad \xi = \begin{bmatrix} \nu \\ \omega \end{bmatrix}
\]

\[ \text{Rank}(A) = 6 \Rightarrow \text{Regular localization} \]

\[ \text{Ker}(A) = \text{Lie algebra of } G_0 \]

\[ \text{Ker}(A)^\perp = m_0 \]

\[ \Rightarrow \xi = A^+b \]
Outline

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   2.4 Analytic results
   2.5 Performance evaluation
   2.6 Symmetric localization
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   2.8 The hybrid algorithm
2. 8 The Hybrid Algorithm

\[ \{y_i\}_{i=1}^{n} \xrightarrow{FSL} g_0 \in SE(3)/G_0 \]

Algorithm: (The Envelopment Algorithm)

**Input:**

a) Meas. data \( \{z_i\}_{i=1}^{m} \);  
b) CAD model and a basis \( (\hat{\eta}_1, ..., \hat{\eta}_r) \) for \( g_0 \);  
c) \( g_0 \in SE(3)/G_0 \) from the FSL algorithm;

**Output:** Optimal solution \( g^* \in SE(3) \)

**Step 0:**

a) Set \( k = 0 \) and \( g^0 = g_0 \);  
b) Compute \( \omega_i^0 \) and \( n_i^0 \), \( i = 1, ..., m \);  
c) Calculate \( \varepsilon^0_c = \sum_i <(g_0)^{-1}z_i - \omega_i^0, n_i^0>^2 \);

**Step 1:**

a) Let \( g^{k+1} = g^k e^{\hat{\lambda}}, \hat{\lambda} \in g_0 \), and solve for \( \lambda \in \mathbb{R}^r \);  
b) Solve for \( \omega_i^{k+1} \) and \( n_i^{k+1} \), \( i = 1, ..., m \);  

\[ g^{k+1} = g^k e^{\hat{\lambda}} \]
c) Calculate $\varepsilon_{e}^{k+1}$;
d) If $(1 - \varepsilon_{e}^{k+1} / \varepsilon_{e}^{k}) < \varepsilon$ and $\langle (g^{k+1})^{-1} z_{i} - \omega_{i}^{k+1}, n_{i}^{k+1} \rangle \geq \delta_{i}$, then report the solution $g^{*} = g^{k+1}$; else, set $k = k + 1$;
e) If $k \leq K_{0}$, then go to step 1(a); else, exit.

Example 1:

$S_{1}$: Finished surface

$G_{0} = \{e^{(\lambda_{1} \hat{\xi}_{1} + \lambda_{2} \hat{\xi}_{2} + \lambda_{3} \hat{\xi}_{3})} | \lambda_{i} \in R\}$

$M_{0} = \text{span}\{\hat{\xi}_{3}, \hat{\xi}_{4}, \hat{\xi}_{5}\}$

$\overline{S}_{1}, \overline{S}_{2}, \overline{S}_{3}$: Unfinished surface
Example 2:

\( S_1, S_2 \): Finished surface

\[ G_0 = \{ e^{\lambda \xi_3} \mid \lambda \in \mathbb{R} \} \]

\[ M_0 = \{ \xi_1, \xi_2, \xi_4, \xi_5, \xi_6 \} \]

\( \overline{S}_1 \): Unfinished surface
A7. Tolerance formulation and verification
ANSI Y14.5 standard

- **Form, profile** ------- 5 cases
  - Flatness
  - Straightness
  - Cylindricity
  - Circularity
  - Profile

- **Orientation** ------- 3 cases
  - Planar orientation
  - Cylindrical orientation
  - Linear orientation

- **Location** ------- >=52 cases
- **Runout** ------- 4 cases

**Problems:**
- graphic expression;
- case by case basis.
- effic. but inconsist. LSA

**Diff. geometric tools**
- Unified formulation; (simple and compact)
- Efficient verification.

ANSI Y14.5.1M partially addressed those problems but not completely.
Example: flatness tolerance

\[ p: \text{ Given } Y = \{y_i\}, \text{ CAD model, find } g \in SE(3)/G_0, \text{ s.t.} \]
\[ l = \min_{g \in SE(3)/G_0} (\max_{y_i \in Y} d(g, y_i) - \min_{y_i \in Y} d(g, y_i)) \]

\[ p': \text{ argmin}_{g \in SE(3)/G_0} f(g, s_1, s_2) = s_2 - s_1 \]
\[ \text{subject to } s_1 \leq d(g, u_i) \leq s_2 \]
<table>
<thead>
<tr>
<th>Features</th>
<th>Symbols</th>
<th>Symmetry group</th>
<th>Configuration Space Q</th>
<th>Description</th>
<th>Dimension</th>
<th>Applicable form tolerances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle in $\mathbb{R}^2$</td>
<td><img src="image" alt="Circle" /></td>
<td>SO(2)</td>
<td>SE(2)/$G_0$</td>
<td>Rotation about the center</td>
<td>2</td>
<td>Circularity</td>
</tr>
<tr>
<td>Straight line in $\mathbb{R}^2$</td>
<td><img src="image" alt="Line" /></td>
<td>T(1)</td>
<td>SE(2)/$G_0$</td>
<td>Translation along the line</td>
<td>2</td>
<td>Straightness</td>
</tr>
<tr>
<td>Sphere</td>
<td><img src="image" alt="Sphere" /></td>
<td>SO(3)</td>
<td>SE(3)/$G_0$</td>
<td>Rotation about the center</td>
<td>3</td>
<td>Sphericity, Circularity</td>
</tr>
<tr>
<td>Plane</td>
<td><img src="image" alt="Plane" /></td>
<td>SE(2)</td>
<td>SE(3)/$G_0$</td>
<td>Rotation about the normal &amp; translation on the plane</td>
<td>3</td>
<td>Flatess, Straightness</td>
</tr>
<tr>
<td>Straight line in $\mathbb{R}^3$</td>
<td><img src="image" alt="Line" /></td>
<td>SO(2)×T(1)</td>
<td>SE(3)/$G_0$</td>
<td>Rotation about &amp; translation along the line</td>
<td>4</td>
<td>Straightness</td>
</tr>
<tr>
<td>Cylinder</td>
<td><img src="image" alt="Cylinder" /></td>
<td>SO(2)×T(1)</td>
<td>SE(3)/$G_0$</td>
<td>Rotation about &amp; translation along the axis</td>
<td>4</td>
<td>Cylindricity, Straightness</td>
</tr>
<tr>
<td>Cone (resolved surface)</td>
<td><img src="image" alt="Cone" /></td>
<td>SO(2)</td>
<td>SE(3)/$G_0$</td>
<td>Rotation about the axis</td>
<td>5</td>
<td>Conicity, Circularity</td>
</tr>
<tr>
<td>Tabular surface</td>
<td><img src="image" alt="Tabular Surface" /></td>
<td>T(1)</td>
<td>SE(3)/$G_0$</td>
<td>Translation along a line</td>
<td>5</td>
<td>Profile, Straightness</td>
</tr>
<tr>
<td>Sculptured surface</td>
<td><img src="image" alt="Sculptured Surface" /></td>
<td>I</td>
<td>SE(3)</td>
<td>Identity</td>
<td>6</td>
<td>Profile</td>
</tr>
</tbody>
</table>
Table 2: Flatness evaluation.

<table>
<thead>
<tr>
<th>Method \ Performance</th>
<th>Example 1 (n = 15)</th>
<th>Example 2 (n = 25)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tolerance Zone</td>
<td>Computation Time(s)/Iter.</td>
</tr>
<tr>
<td>Minimum Zone</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SMZ</td>
<td>1.96116</td>
<td>0.048/12</td>
</tr>
<tr>
<td>CaFeli</td>
<td>1.96116</td>
<td>0.030/3</td>
</tr>
<tr>
<td>Wang</td>
<td>1.96116</td>
<td>0.200/9</td>
</tr>
<tr>
<td>Cvx hull[i8]</td>
<td>2.00000</td>
<td>N/A</td>
</tr>
<tr>
<td>Least Squares</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forbes[i6]</td>
<td>2.53212</td>
<td>0.01/-</td>
</tr>
<tr>
<td>FSL[i8]</td>
<td>2.53213</td>
<td>0.009/9</td>
</tr>
<tr>
<td>FSL/SMZ</td>
<td>1.96116</td>
<td>0.041/(9+8)</td>
</tr>
</tbody>
</table>

Table 3: Circularity evaluation

<table>
<thead>
<tr>
<th>Method \ Performance</th>
<th>Example 1 (n = 8)</th>
<th>Example 2 (n = 24)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tolerance Zone</td>
<td>Computational Time(s)/Iter.</td>
</tr>
<tr>
<td>Minimum Zone</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SMZ</td>
<td>2.243271</td>
<td>0.012/4</td>
</tr>
<tr>
<td>Wang</td>
<td>2.243271</td>
<td>0.100/8</td>
</tr>
<tr>
<td>Least Squares</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forbes[i6]</td>
<td>2.530115</td>
<td>0.006/2</td>
</tr>
<tr>
<td>FSL[i8]</td>
<td>2.530117</td>
<td>0.003/6</td>
</tr>
<tr>
<td>FSL/SMZ</td>
<td>2.243271</td>
<td>0.015/(6+4)</td>
</tr>
</tbody>
</table>

Outline

1. Motivation
2. Geometric algorithms for workpiece localization
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6. Implementation and applications
7. Conclusion
3. Reliability Analysis

• Q: With measurement errors and a finite no. of sampling points, how reliable is the computed solution?

• Example:

Orientation:

\[ \Delta \theta \propto \frac{1}{d} \]

Not too reliable

Translation:

\[ \text{error} \propto \frac{1}{\sin \Phi} \]

Not reliable in x-direction

More reliable
\{y_i\}_{i=1}^n \rightarrow g^* = (R^*, p^*) \xrightarrow{\text{Estimate of}} g_a = (R_a, p_a)

\varepsilon^* = \sum_{i=1}^n <g^* y_i - x_i^*, n_i^*>^2 \leq \varepsilon_a = \sum_{i=1}^n <g_a y_i - x_i^*, n_i^*>^2

Assume:
\varepsilon_a = \varepsilon_1 + \varepsilon_*

and
\begin{align*}
<g^* y_i - x_i^*, n_i^*>^2, & \quad i = 1, \ldots, n \\
<g_a y_i - x_i^*, n_i^*>^2, & \quad i = 1, \ldots, n
\end{align*}

are normally distributed, with variance 
\varepsilon_* & \varepsilon_a, \text{ respectively}

\Rightarrow F = \frac{\varepsilon_a}{\varepsilon_*} : F - \text{distribution}

l = n - 6(\text{dof})

Let \(F_{\varepsilon}(l, l)\) be critical value at the \(\varepsilon\)-level corresponding to \(\text{dof}(l, l)\).
\[ P(F > F_{\varepsilon(l,l)}) = \varepsilon \]

or

\[ P(F < F_{\varepsilon(l,l)}) = 1 - \varepsilon \]

The probability that \( F = (\varepsilon_* + \varepsilon_1) / \varepsilon_* < F_{\varepsilon(l,l)} \) is equal to \((1 - \varepsilon)\).

**Translational Reliability**

Let \( \delta_p = (\delta_{p_x}, \delta_{p_y}, \delta_{p_z})^T \), and

\[ \delta = \sqrt{\delta_{p_x}^2 + \delta_{p_y}^2 + \delta_{p_z}^2} \]

\[
\varepsilon_p = \delta_p \cdot \begin{bmatrix} n_1^T \ldots n_n^T \end{bmatrix} \cdot \delta_p^T \cdot J_p \cdot \delta_p
\]

\[ \varepsilon_a = \varepsilon_* + \varepsilon_p \]
The probability that
\[ \frac{\varepsilon_p}{\varepsilon_*} < (F_{\varepsilon(l,l)} - 1) \]
is equal to \((1 - \varepsilon)\)

Prop: Translational error \(d\) along any direction is bounded by
\[ d \leq ((F_{\varepsilon(l,l)} - 1)\varepsilon_* / \lambda_p)^{1/2} \]
where \(\lambda_p\) is the smallest eigenvalue of \(J_p\).
• Rotational Reliability:

Assume $||\omega||=1$, and

$$R_* = e^{\hat{\omega}\theta}R_a \cong (I + \hat{\omega}\theta)R_a$$

$$\varepsilon_r = \omega^T \cdot \left[ (n_1 \times q_1), \ldots, (n_n \times q_n) \right]$$

$$\varepsilon_r = \omega^T \cdot \left[ (n_1 \times q_1)^T \right]$$

$$\varepsilon_r = \omega^T \cdot \left[ \omega \cdot \theta^2 \right]$$

Prop: Rotational error $\theta$ along any direction is bounded by

$$\theta \leq \left( (F_{\varepsilon(l,l)} - 1)\varepsilon_* / \lambda_r \right)^{1/2}$$

where $\lambda_p$ is the smallest eigenvalue of $J_r$. 
Outline

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4. Sampling and Probe Radius Compensation

4.1 Discrete symmetry

- touch probe
- non-touch probe

Touch probe is a de facto choice.

- High accuracy
- Easy of use
- Less calibration
What we record is center point set \( \{y_i'\} \).

We need contact point set \( \{y_i\} \) for localization algorithms.

**Significant errors** will be introduced if not compensated since \( r \) is of several mms.

- \( r \): probe radius
- \( y_i \): contact point
- \( n_i \): normal in \( C_W \)
- \( y_i' \): probe center point
• Compensation – Our Proposed Method

Note:

\[ y'_i = y_i + rn'_i \]
\[ x'_i = x_i + rn_i \]
\[ n'_i = gn_i \]

- \( y_i \): contact point in \( C_W \)
- \( y'_i \): probe center point in \( C_W \)
- \( x_i \): contact point in \( C_M \)
- \( x'_i \): probe center point in \( C_M \)
- \( r \): probe radius
- \( n_i \): normal in \( C_M \)
- \( n'_i \): normal in \( C_W \)
The objective function becomes

$$
\varepsilon(g, x_1, \cdots, x_n) = \sum_{i=1}^{n} \| y_i - gx_i \|^2
$$

$$
= \sum_{i=1}^{n} \| (y_i + rn_i') - (gx_i + rn_i') \|^2
$$

$$
= \sum_{i=1}^{n} \| y_i' - gx_i' \|^2
$$

\{y'_i\} and \{x'_i\} lie on offset surfaces of the original ones

⇒ existing algorithms can be used to solve for g using \{y'_i\}.
4.2 Sampling

Q: How many points should be probed?
   ▪ For a given number of points, where to probe?

Our computer-aided probing strategy uses:

▪ Reliability analysis to determine if the probed points are adequate
▪ Sequential optimal planning to determine the locations where probing are to take place
Recall the objective function

\[
\varepsilon(g, x_1, \cdots, x_n) = \sum_{i=1}^{n} \left\| y_i - gx_i \right\|^2 = \sum_{i=1}^{n} \left\| g^{-1}y_i - x_i \right\|^2
\]

Let \( g^* \) be the optimal solution of localization algorithms
\( x_i^* \) be the optimal solution of home points
\( g_a \) be the actual transformation between \( C_M \) and \( C_W \)

It is easy to see

\[
\varepsilon_a = \sum_{i=1}^{n} \left\| g_a^{-1}y_i - x_i^* \right\|^2 \geq \varepsilon_* = \sum_{i=1}^{n} \left\| g^{-1}y_i - x_i^* \right\|^2
\]

If we assume that sampling errors are normally distributed,

\( g_a^{-1}y_i - x_i^* \) and \( g^{-1}y_i - x_i^* \) are normally distributed.
Theorem 1:
If $X$ is normally distributed with mean $\mu$ and variance $\sigma^2$, and $X_1, \ldots, X_n$ is a random sample of size $n$ of $X$, then the random variable
$$U = \sum_{i=1}^{n} (X_i - \mu)^2 / \sigma^2$$
will possess a chi-square distribution with $n$ degrees of freedom.

Theorem 2:
If $U$ and $V$ possess independent chi-square distributions with $v_1$ and $v_2$ degrees of freedom, respectively, then
$$F = \frac{U/v_1}{V/v_2}$$
has the $F$ distribution with $v_1$ and $v_2$ degrees of freedom given by
$$f(F) = c F^{\frac{1}{2}(v_1-2)} (v_2 + v_1 F)^{-\frac{1}{2}(v_1+v_2)}$$
where $c$ is a constant related with $v_1$ and $v_2$ only.

$$F = \frac{\varepsilon_a}{\varepsilon_*}$$
is a $F$ distribution.
By previous research, we define

\[
N_p = \begin{bmatrix}
    n_1^T \\
n_2^T \\
    \vdots \\
n_n^T
\end{bmatrix}
\quad
N_r = -\begin{bmatrix}
    (n_1 \times q_1)^T \\
    (n_2 \times q_2)^T \\
    \vdots \\
    (n_n \times q_n)^T
\end{bmatrix}
\quad
J_p = N_p^T N_p
\quad
J_r = N_r^T N_r
\]

where \( q_i \) is the \( i^{th} \) home point and \( n_i \) is the corresponding normal vector.

Translation error \( d \) along any direction is bounded by

\[
d \leq ((F_{\varepsilon(l,l)} - 1) \varepsilon_*/\lambda_p)^{1/2}
\]

smallest eigenvalue of \( J_p \)

Rotation error \( \theta \) along any direction is bounded by

\[
\theta \leq ((F_{\varepsilon(l,l)} - 1) \varepsilon_*/\lambda_r)^{1/2}
\]

smallest eigenvalue of \( J_r \)

\( F_{\varepsilon(l,l)} \): the critical value at the \( \varepsilon \)-level of the degrees of freedom \((l,l)\)

\( \varepsilon \): the confidence limit

\( l = n-6 \) is the degree of freedom
Why the locations of measurement points are important?

For 3D sculptured object, human intuition does not work well!
**Computer-Aided Probing Strategy – Fixture Model**

From fixture planning, we have

\[ \delta y_i = - [n_i^T \quad (r_i \times n_i)^T] \begin{bmatrix} \nu \\ \omega \end{bmatrix} = h_i^T \delta \xi \]

the \( i \text{th} \) locator error \quad workpiece location error

Combine equations at all locators,

\[
\delta y = \begin{bmatrix} \delta y_1 \\ \delta y_2 \\ \vdots \\ \delta y_n \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & \cdots & h_n \end{bmatrix}^T \delta \xi = G^T \delta \xi
\]

Note:

\[
\|\delta y\| = \delta y^T \delta y = \delta \xi^T G G^T \delta \xi = \delta \xi^T M \delta \xi
\]

the matrix \( M \) relates locator errors with workpiece location errors
Computer-Aided Probing Strategy – Optimal Planning

We follow the D-optimization (Wang 00) with the index

$$\max \det(M)$$

(In a point set domain)

Notice that

$$M = GG^T = \sum_{i=1}^{n} h_i h_i^T$$

If M contains n locators, we delete one from the n locators, then

$$M_j = M - h_j h_j^T$$

furthermore

$$\det(M_j) = (1 - p_{jj}) \det(M)$$

$$p_{jj} = h_j^T M^{-1} h_j$$

and

$$M_j^{-1} = M^{-1} + (M^{-1} h_j)(M^{-1} h_j)^T / (1 - p_{jj})$$

By minimizing $p_{jj}$, we can sequentially optimize the index.

sequential deletion method
Computer-Aided Probing Strategy – The Strategy

Input: CAD model of the workpiece, $\alpha_r^d$, $\alpha_p^d$, $\varepsilon$

- with $N'$ discretized points
- acceptable translation error bound
- acceptable rotation error bound
- the confidence limit

Output: estimated transformation $g$ within error bounds

- Manually probe 7 points ($n=7$)

Reliability analysis

- Two error bounds are satisfied
- Not satisfied

- Set $n=n+k$

- $n>N$?
  - Yes
    - Error
  - No
    - Sequential planning

Probing

Success

$N \leq N'$

Candidate probing points set
Simulating Aided Probing Strategy – Simulation

Simulation model

Simulation setup:

- $\alpha_p^d = 0.1\,mm$, $\alpha_r^d = 0.1\,\text{deg}$, $\varepsilon = 95\%$
- Given $g_d$, normally distributed noise introduced
- PII 400 PC
- Two sequential optimal planning algorithms

$N' = 1599$

$N = 991$
Sequential optimal planning:

Sequential deletion algorithm:
we get final 6 points planning with 69.26s in MATLAB. $\det(M) = 7.056 \times 10^{11}$

Sequential addition algorithm:

1. Get 6 points maxdet(M) planning
   - Random generation of 6 points (G full rank)
   - Improve by interchange
   
   Interchange a current point $j$ and a candidate point $k$,
   $$\det(M_{jk}) = p_{jk}^2 \det(M)$$
   $$p_{jk} = h_j^T M^{-1} h_k$$
   Maximize $p_{jk}$, we maximize det(M) with one interchange

   we get final 6 points planning with 0.05s in MATLAB. $\det(M) = 9.180 \times 10^{11}$

2. Add point one by one
   Need averagely 0.066s in MATLAB.
Simulation with deletion sequence, with $\mu = 0.01$, $\sigma^2 = 0.01$, $\varepsilon = 95\%$

<table>
<thead>
<tr>
<th>Point number $n$</th>
<th>85</th>
<th>90</th>
<th>95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translation error bound (mm)</td>
<td>0.1003</td>
<td>0.1004</td>
<td>0.0983</td>
</tr>
<tr>
<td>Rotation error bound (degree)</td>
<td>0.1004</td>
<td>0.0968</td>
<td>0.0980</td>
</tr>
</tbody>
</table>

Succeed with 95 points!
Simulation with addition sequence, with $\mu = 0.01$, $\sigma^2 = 0.01$, $\epsilon = 95\%$

<table>
<thead>
<tr>
<th>Point number $n$</th>
<th>215</th>
<th>220</th>
<th>225</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translation error bound (mm)</td>
<td>0.0977</td>
<td>0.0964</td>
<td>0.0945</td>
</tr>
<tr>
<td>Rotation error bound (degree)</td>
<td>0.1014</td>
<td>0.1005</td>
<td>0.099</td>
</tr>
</tbody>
</table>

Succeed with 225 points!
Comparison of $\sigma^2 = 0.01$ and $\sigma^2 = 0.02$

Comparison of $\epsilon = 95\%$ and $\epsilon = 99\%$
Outline

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5. Implementation – The System Integration

CAS system:

- Open architecture CNC machine
- Conventional CNC machine

Two CAS systems
Common Parts:

- **Graphics User Interface**
  - Model viewing control (Compatible with other CAD software)
  - Surface selection for surface probing
  - Visual manipulation of probed points

- **Probe System**

- **Algorithms**
  - Workpiece localization
  - Online compensation
  - Probing control
  - Optimal planning etc.
**Implementation – The System Integration**

Open architecture system:

- CNC Machine
- Software Module
- Host Computer
- Motion Controller
- Machine Table

Conventional system:

- CNC Machine
- Software Module
- Host Computer
- Motion Controller
- Machine Table

**Serial port command (DNC)**
• **Implementation – Experiments**

Several functions:

- Manual Probing
- Auto Probing
- Computer Aided Probing

Options:

\[
\alpha_p^d = 0.1 \text{mm} \quad \alpha_r^d = 0.2 \text{deg} \quad \varepsilon = 95\%
\]
Experimental model

Succeed with 185 points!

N' = 1661

N = 1122
Video Show
Video Show
6. Conclusions

Three important components of building a CAS system have been discussed.

- Robust workpiece localization algorithms
- Accurate probe radius compensation method
- Computer-aided probing strategy

On the basis of these algorithms, two CAS systems have been built.

Simulation and experimental results show that the system is suitable for real-time implementation in manufacturing process.
Reference

Regular Localization:


Symmetric Localization:


Hybrid Localization:


Others:

